CHAPTER 6: CAPITAL ALLOCATION TO RISKY ASSETS

Solutions to Suggested Problems

4. a. The expected cash flow is: \((0.5 \times 70,000) + (0.5 \times 200,000) = 135,000\).

With a risk premium of 8% over the risk-free rate of 6%, the required rate of return is 14%. Therefore, the present value of the portfolio is:

\[
\frac{135,000}{1.14} = 118,421
\]

b. If the portfolio is purchased for $118,421 and provides an expected cash inflow of $135,000, then the expected rate of return \(E(r)\) is as follows:

\[
\frac{(135,000 - 118,421)}{118,421} = 0.14 \text{ or } 14\%
\]

c. If the risk premium over T-bills is now 12%, then the required return is:

\[
6\% + 12\% = 18\%
\]

The present value of the portfolio is now:

\[
\frac{135,000}{1.18} = 114,407
\]

d. For a given expected cash flow, portfolios that command greater risk premiums must sell at lower prices. The extra discount from expected value is a penalty for risk.

Note that in this problem, the investor is the same, i.e., “you”. Therefore, risk aversion is the same in part (a) and part (c). Higher required risk premium by the same investor implies that the investment has become riskier for some reason.

5. When we specify utility by \(U = E(r) - 0.5A\sigma^2\), the utility level for T-bills is: 0.07

The utility level for the risky portfolio is:

\[
U = 0.12 - 0.5 \times A \times (0.18)^2 = 0.12 - 0.0162 \times A
\]

In order for the risky portfolio to be preferred to bills, the following must hold:

\[
0.12 - 0.0162 \times A > 0.07 \Rightarrow A < \frac{0.05/0.0162}{3.09} = 3.09
\]

A must be less than 3.09 for the risky portfolio to be preferred to bills.
10. The portfolio expected return and variance are computed as follows:

\[
W_{\text{bills}} \cdot r_{\text{bills}} + W_{\text{index}} \cdot r_{\text{index}} = r_{\text{portfolio}} = (1) \times (2) + (3) \times (4)
\]

\[
\sigma^2_{\text{Portfolio}} = (3) \times 20\%
\]

<table>
<thead>
<tr>
<th>(W_{\text{bills}})</th>
<th>(r_{\text{bills}})</th>
<th>(W_{\text{index}})</th>
<th>(r_{\text{index}})</th>
<th>(r_{\text{portfolio}})</th>
<th>(\sigma^2_{\text{Portfolio}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05</td>
<td>1</td>
<td>0.13</td>
<td>0.13</td>
<td>0.2</td>
</tr>
<tr>
<td>0.2</td>
<td>0.05</td>
<td>0.8</td>
<td>0.13</td>
<td>0.114</td>
<td>0.16</td>
</tr>
<tr>
<td>0.4</td>
<td>0.05</td>
<td>0.6</td>
<td>0.13</td>
<td>0.098</td>
<td>0.12</td>
</tr>
<tr>
<td>0.6</td>
<td>0.05</td>
<td>0.4</td>
<td>0.13</td>
<td>0.082</td>
<td>0.08</td>
</tr>
<tr>
<td>0.8</td>
<td>0.05</td>
<td>0.2</td>
<td>0.13</td>
<td>0.066</td>
<td>0.04</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0</td>
<td>0.13</td>
<td>0.05</td>
<td>0</td>
</tr>
</tbody>
</table>

11. Computing utility from \(U = E(r) - 0.5 \times A \sigma^2 = E(r) - \sigma^2\), we arrive at the values in the column labeled \(U(A = 2)\) in the following table:

\[
U(A = 2) = E(r) - 0.5 \times \sigma^2
\]

<table>
<thead>
<tr>
<th>(W_{\text{bills}})</th>
<th>(W_{\text{index}})</th>
<th>(r_{\text{portfolio}})</th>
<th>(\sigma^2_{\text{portfolio}})</th>
<th>(U(A = 2))</th>
<th>(U(A = 3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.13</td>
<td>0.2</td>
<td>0.04</td>
<td>0.0900</td>
</tr>
<tr>
<td>0.2</td>
<td>0.8</td>
<td>0.114</td>
<td>0.16</td>
<td>0.0256</td>
<td>0.0884</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>0.098</td>
<td>0.12</td>
<td>0.0144</td>
<td>0.0836</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4</td>
<td>0.082</td>
<td>0.08</td>
<td>0.0064</td>
<td>0.0756</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2</td>
<td>0.066</td>
<td>0.04</td>
<td>0.0016</td>
<td>0.0644</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>0.0500</td>
</tr>
</tbody>
</table>

The column labeled \(U(A = 2)\) implies that investors with \(A = 2\) prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.

12. The column labeled \(U(A = 3)\) in the table above is computed from:

\[
U = E(r) - 0.5A \sigma^2 = E(r) - 1.5\sigma^2
\]

The more risk averse investors prefer the portfolio that is invested 60% in the market, rather than the 100% market weight preferred by investors with \(A = 2\).

13. Expected return = \((0.7 \times 18\%) + (0.3 \times 8\%) = 15\%

Standard deviation = \(0.7 \times 28\% = 19.6\%

14. Investment proportions:

- 30.0% in T-bills
- \(0.7 \times 25\% = 17.5\%\) in Stock A
- \(0.7 \times 32\% = 22.4\%\) in Stock B
- \(0.7 \times 43\% = 30.1\%\) in Stock C
15. Your reward-to-volatility ratio: \[ S = \frac{0.18 - 0.08}{0.28} = 0.3571 \]

Client's reward-to-volatility ratio: \[ S = \frac{0.15 - 0.08}{0.196} = 0.3571 \]

16. 

![Graph of CAL (Slope = 0.3571)](image)

17. a. \[ E(r_c) = r_f + y \times [E(r_P) - r_f] = 8\% + y \times (18\% - 8\%) \]

If the expected return for the portfolio is 16\%, then:

\[ 16\% = 8\% + 10\% \times y \implies y = \frac{0.16 - 0.08}{0.10} = 0.8 \]

Therefore, in order to have a portfolio with expected rate of return equal to 16\%, the client must invest 80\% of total funds in the risky portfolio and 20\% in T-bills.

b. 

<table>
<thead>
<tr>
<th>Client’s Investment Proportions</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.00% in T-bills</td>
<td></td>
</tr>
<tr>
<td>0.80 \times 25% = 20.00%</td>
<td>in Stock A</td>
</tr>
<tr>
<td>0.80 \times 32% = 25.60%</td>
<td>in Stock B</td>
</tr>
<tr>
<td>0.80 \times 43% = 34.40%</td>
<td>in Stock C</td>
</tr>
</tbody>
</table>
c. \( \sigma_C = 0.80 \times \sigma_P = 0.80 \times 28\% = 22.4\% \)

18. a. Note that \( \sigma_C = y \times 28\% \)

The constraint is: \( \sigma_C \leq 18\% \)

\[ \rightarrow y \times 28\% \leq 18\% \]

\[ \rightarrow y \leq \frac{18}{28} = 0.64 = 64\% \]

The constrained optimal solution should be as close as possible to the unconstrained optimal solution. Since unconstrained solution for optimal \( y \) was 80\%, the constrained solution will be 64\%. So, \( y = 64\% \).

b. \( E(r_C) = 0.36 \times 8\% + 0.64 \times 18\% = 14.4\% \)

19. a. \( y^* = \frac{E(r_p) - r_f}{\sigma_P^2} = \frac{0.18 - 0.08}{3.5 \times 0.28^2} = \frac{0.10}{0.2744} = 0.3644 \)

Therefore, the client’s optimal proportions are: 36.44\% invested in the risky portfolio and 63.56\% invested in T-bills.

b. \( E(r_C) = 0.6356 \times 8\% + 0.3644 \times 18\% = 11.64\% \)
\( \sigma_C = 0.3644 \times 28\% = 10.20\% \)

21. a. \( E(r_C) = 8\% = 5\% + y \times (11\% - 5\%) \Rightarrow y = \frac{0.08 - 0.05}{0.11 - 0.05} = 0.5 \)

Your client should invest 50\% of her total investment budget in the risky portfolio and 50\% in the risk-free asset.

b. \( \sigma_C = y \times \sigma_P = 0.50 \times 15\% = 7.50\% \)

c. The first client is more risk averse, preferring investments that have less risk as evidenced by the lower standard deviation.
22. Johnson requests the portfolio standard deviation to equal one half the market portfolio standard deviation. The market portfolio $\sigma_M = 20\%$, which implies $\sigma_p = 10\%$. The intercept of the CML equals $r_f = 0.05$ and the slope of the CML equals the Sharpe ratio for the market portfolio (35%). Therefore using the CML:

$$E(r_p) = r_f + \frac{E(r_M) - r_f}{\sigma_M} \sigma_p = 0.05 + 0.35 \times 0.10 = 0.085 = 8.5\%$$

23. Data: $r_f = 5\%$, $E(r_M) = 13\%$, $\sigma_M = 25\%$, and $r^B = 9\%$

The CML and indifference curves are as follows:

![CML and indifference curves diagram]

24. For $y$ to be less than 1.0 (that the investor is a lender), risk aversion ($A$) must be large enough such that:

$$y = \frac{E(r_M) - r_f}{A\sigma_m^2} < 1 \Rightarrow A > \frac{0.13 - 0.05}{0.25^2} = 1.28$$

For $y$ to be greater than 1 (the investor is a borrower), $A$ must be small enough:

$$y = \frac{E(r_M) - r^B_M}{A\sigma_M^2} > 1 \Rightarrow A < \frac{0.13 - 0.09}{0.25^2} = 0.64$$

For values of risk aversion within this range, the client will neither borrow nor lend but will hold a portfolio composed only of the optimal risky portfolio:

$$y = 1 \text{ for } 0.64 \leq A \leq 1.28$$
Solutions to Suggested CFA Problems

1. Utility for each investment = $E(r) - 0.5 \times 4 \times \sigma^2$

   We choose the investment with the highest utility value, Investment 3.

   \[
   \begin{array}{c|c|c|c}
       \text{Investment} & \text{Expected return $E(r)$} & \text{Standard deviation $\sigma$} & \text{Utility $U$} \\
       \hline
       1 & 0.12 & 0.30 & -0.0600 \\
       2 & 0.15 & 0.50 & -0.3500 \\
       3 & 0.21 & 0.16 & 0.1588 \\
       4 & 0.24 & 0.21 & 0.1518 \\
   \end{array}
   \]

2. When investors are risk neutral, then $A = 0$; the investment with the highest utility is Investment 4 because it has the highest expected return.

3. (b)

4. Indifference curve 2 because it is tangent to the CAL.

5. Point E

6. $(0.6 \times $50,000) + [0.4 \times (-$30,000)] - $5,000 = $13,000

7. (b) Higher borrowing rates will reduce the total return to the portfolio and this results in a part of the line that has a lower slope.

8. Expected return for equity fund = T-bill rate + Risk premium = 6% + 10% = 16%

   Expected rate of return of the client’s portfolio = (0.6 × 16%) + (0.4 × 6%) = 12%

   Expected return of the client’s portfolio = $0.12 \times 100,000 = $12,000

   (which implies expected total wealth at the end of the period = $112,000)

   Standard deviation of client’s overall portfolio = 0.6 × 14% = 8.4%

9. Reward-to-volatility ratio = $\frac{10}{0.14} = 0.71$