Chapter 9

The Capital Asset Pricing Model

Capital Asset Pricing Model (CAPM)

- The Capital Asset Pricing Model, almost always referred to as the CAPM, is a centerpiece of modern financial economics.

- The model gives us a precise prediction of the relationship that we should observe between the risk of an asset and its expected return.

- This relationship serves two vital functions.

Capital Asset Pricing Model (CAPM)

- First, it provides a benchmark rate of return for evaluating possible investments – what the "fair" return of a security is given its risk.
- Second, the model helps us to make an educated guess as to the expected return on assets that have not yet been traded in the marketplace.
- Although the CAPM does not fully withstand empirical tests, it is widely used because of the insight it offers and because its accuracy is deemed acceptable for important applications.

Capital Asset Pricing Model (CAPM)

- The Capital Asset Pricing Model is a set of predictions concerning equilibrium expected returns on risky assets.
- Harry Markowitz laid down the foundation of modern portfolio management in 1952.
- The CAPM was published 12 years later in articles by William Sharpe, John Lintner, and Jan Mossin.
- The CAPM is derived using principles of diversification with simplified assumptions.

Assumptions

1. <u>Individual behavior</u>

a. Investors are rational, mean-variance optimizers.

b. Their planning horizon is a single period.

c. Investors have homogeneous expectations (identical input lists).

Assumptions

2. <u>Market structure</u>

a. All assets are publicly held and trade on public exchanges, short positions are allowed, and investors can borrow or lend at a common risk-free rate.

b. All information is publicly available.

c. No taxes.

d. No transaction costs.

Resulting Equilibrium Conditions

1. All investors will hold the same portfolio for risky assets – market portfolio.

2. Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value.

Resulting Equilibrium Conditions

3. Risk premium on the market depends on the average risk aversion of all market participants.

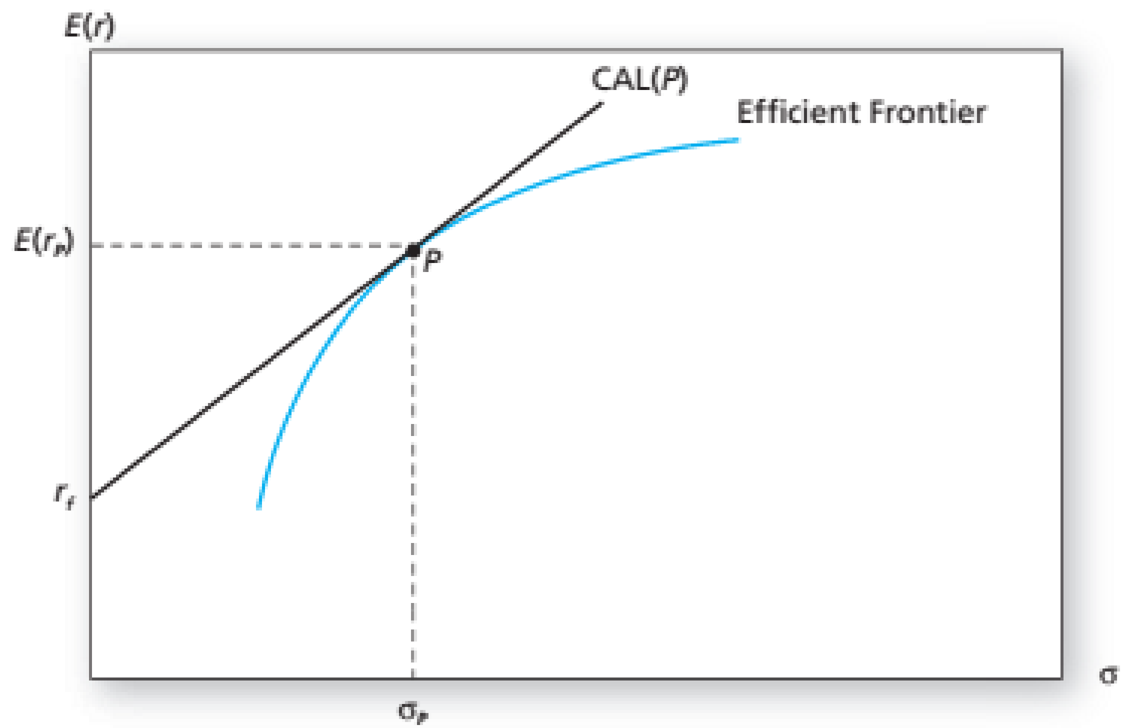
4. Risk premium on an individual security is a function of its covariance with the market.

Equilibrium Conditions

1. All investors will hold the same portfolio for risky assets – market portfolio.

Suppose all investors optimized their portfolios á la Markowitz. That is, each investor uses an input list to draw an efficient frontier employing all available risky assets and identifies an efficient risky portfolio, *P*, by drawing the tangent CAL to the frontier as in this Figure. As a result, each investor holds securities in the investable universe with weights arrived at by the Markowitz optimization process.

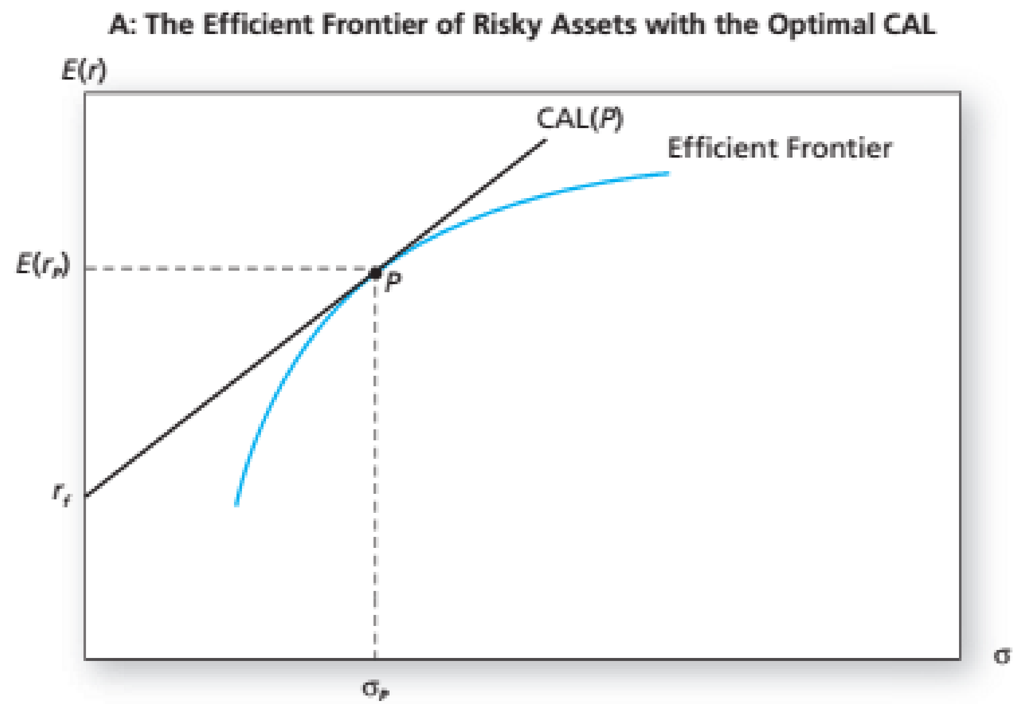
A: The Efficient Frontier of Risky Assets with the Optimal CAL



Equilibrium Conditions

1. All investors will hold the same portfolio for risky assets – market portfolio.

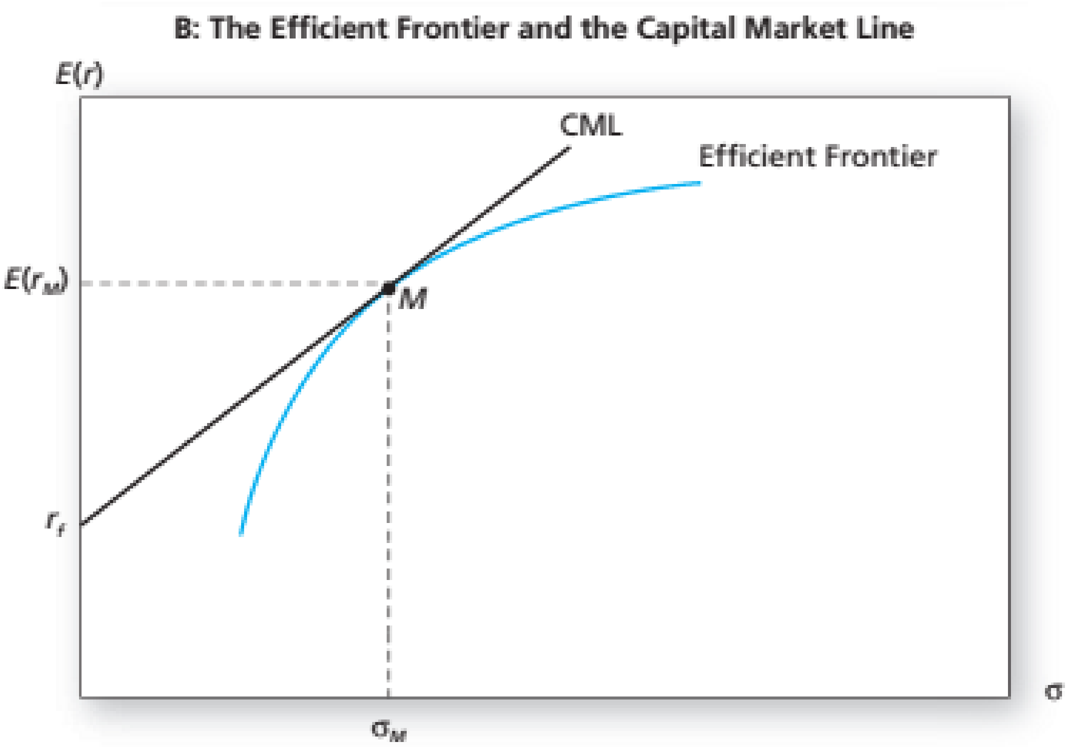
The CAPM asks what would happen if all investors shared an identical investable universe and used the same input list to draw their efficient frontiers. Obviously, their efficient frontiers would be identical. Facing the same risk-free rate, they would then draw an identical tangent CAL and naturally all would arrive at the same risky portfolio, *P*. All investors therefore would choose the same set of weights for each risky asset. What must be these weights?



Equilibrium Conditions

1. All investors will hold the same portfolio for risky assets – market portfolio.

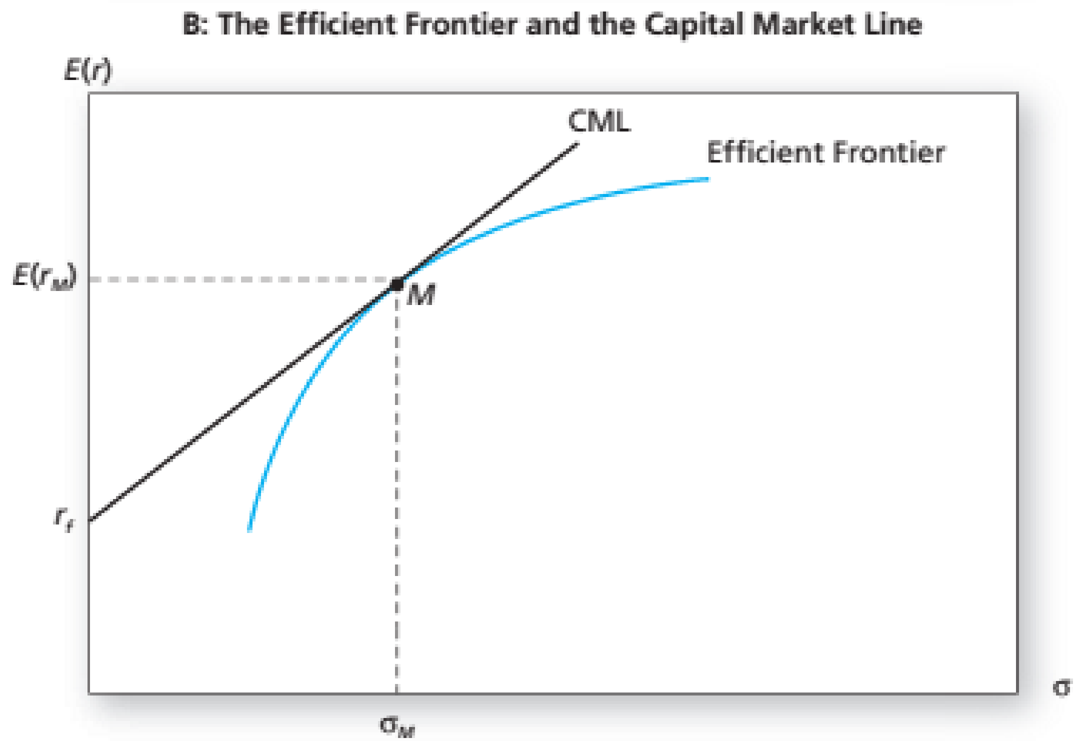
A key insight of the CAPM is this: Because the market portfolio is the aggregation of all of these identical risky portfolios, it too will have the same weights. Therefore, if all investors choose the same risky portfolio, it must be the market portfolio, that is, the value-weighted portfolio of all assets in the investable universe. Therefore, the capital allocation line based on each investor's optimal risky portfolio will in fact also be the capital market line, as depicted in this Figure.



Equilibrium Conditions

1. All investors will hold the same portfolio for risky assets – market portfolio.

When we sum over, or aggregate, the portfolios of all individual investors, lending and borrowing will cancel out (because each lender has a corresponding borrower), and the value of the aggregate risky portfolio will equal the entire wealth of the economy. This is the market portfolio, *M*. The proportion of each stock in this portfolio equals the market value of the stock divided by the sum of the market value of all stocks.



Capital Market Line - Slope and Market Risk Premium

In the CML,

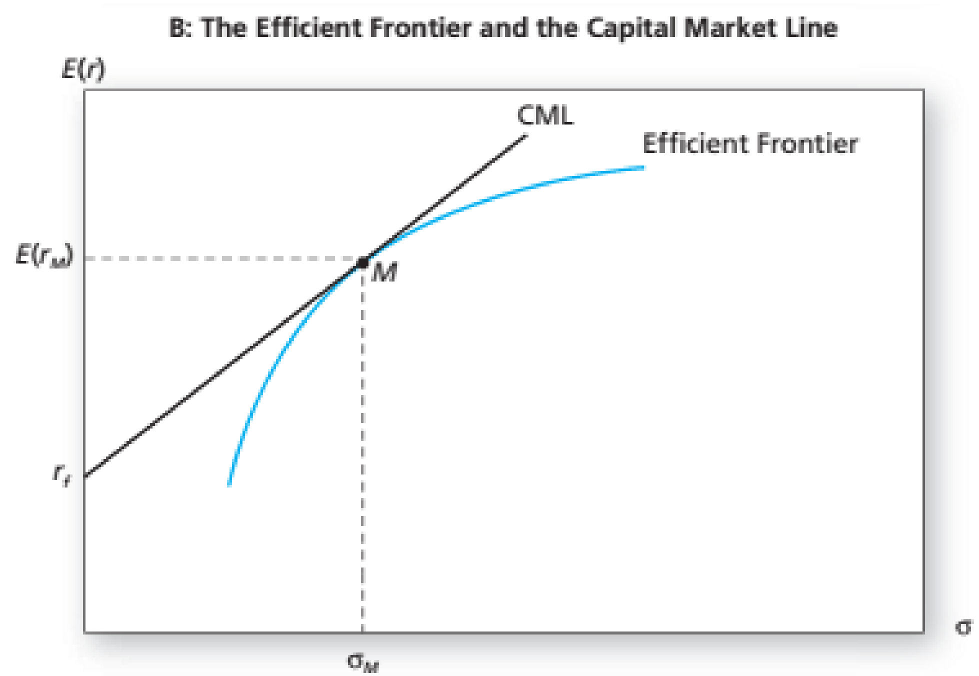
M = Market Portfolio

r_f = The risk free rate

$E(r_M) - r_f$ = Market risk premium

Slope of CML: $\dfrac{E(r_M)-r_f}{\sigma_m}$

The Sharpe ratio of the CML is often referred to as the *market price of risk* – the price that investors demand for assuming risk.



However, to be technically precise, market price of risk is actually denoted by:

$$\frac{E(r_M) - r_f}{\sigma_m^2}$$

Equilibrium Conditions

2. Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value.

- So, why do we say that the optimal *P*, which also turns out to be *M*, that each investor holds must include all securities in the investable universe?

- To answer this question, suppose that the optimal portfolio of our investors does not include the stock of some company, such as Delta Airlines.

- When all investors avoid Delta stock, the demand is zero, and Delta's price takes a free fall.

Equilibrium Conditions

2. Market portfolio contains all securities and the proportion of each security is its market value as a percentage of total market value.

- As Delta stock gets progressively cheaper, it becomes ever more attractive and other stocks look relatively less attractive.

- Ultimately, Delta reaches a price where it is attractive enough to include in the optimal stock portfolio.

- Such a price adjustment process guarantees that all stocks will be included in the optimal portfolio. It shows that all assets have to be included in the market portfolio.

Equilibrium Conditions

3. Risk premium on the market depends on the average risk aversion of all market participants.

- If all investors choose to invest in portfolio *M* and the risk-free asset, what can we deduce about the equilibrium risk premium of portfolio *M*?

- Recall that each individual investor chooses a proportion *y*, allocated to the optimal portfolio *M*, such that:

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2}$$

where $E(r_M) - r_f = E(R_M)$ is the risk premium (expected excess return) on the market portfolio.

Equilibrium Conditions

3. Risk premium on the market depends on the average risk aversion of all market participants.

- In the simplified CAPM economy, risk-free investments involve borrowing and lending among investors. Any borrowing position must be offset by the lending position of the creditor.

- This means that net borrowing and lending across all investors must be zero → For the 'representative' investor, the average position in the risky portfolio is:

$$100\%, \text{ i.e., } \bar{y} = 1$$

Equilibrium Conditions

3. Risk premium on the market depends on the average risk aversion of all market participants.

Substituting the representative investor's risk aversion, $\bar{A}$, for A, setting $y = \bar{y} = 1$, $y = \frac{E(r_M)-r_f}{A\sigma_M^2}$ becomes:

$$\frac{E(r_M) - r_f}{A\sigma_M^2} = 1$$

$$\rightarrow E(r_M) - r_f = E(R_M) = \bar{A}\sigma_M^2$$

Equilibrium Conditions

4. Risk premium on an individual security is a function of its covariance with the market.

- In CAPM, all investors holding a certain proportion of the market portfolio – theoretically the most diversified portfolio that the investable universe offers. As a result, all non-systematic risks or unique risks are diversified away.

- Therefore, for individual securities, in CAPM, the only relevant risk is market risk or non-diversifiable risk.

- The aggregate market risk should be captured by:

    - The volatility of the market portfolio M since it contains all securities in the investable universe. And indeed, aggregate market risk is measured by: σ_M^2

Equilibrium Conditions

4. Risk premium on an individual security is a function of its covariance with the market.

- For an individual security i, the relevant measure of market risk should be a function of how its return moves with the market return, i.e., covariance of the returns of the security and the market portfolio, σ_{im}, which can also be interpreted as security i's contribution to the market risk.

- A basic principle of the CAPM equilibrium is that all investments should offer the same reward-to-risk ratio.

- If the ratio were better for one investment than another, investors would rearrange their portfolios, tilting toward the alternative with the better trade-off and shying away from the other.

- Such activity would impart pressure on security prices until the ratios were equalized.

Equilibrium Conditions

- Therefore, we can conclude that the reward-to-risk ratios of security i and the market portfolio M should be equal:

$$\frac{E(R_i)}{\sigma_{im}} = \frac{E(R_M)}{\sigma_M^2}$$

$$\rightarrow E(r_i) - r_f = \frac{\sigma_{im}}{\sigma_M^2}\left[E(r_m) - r_f\right]$$

$$\rightarrow E(r_i) = r_f + \beta_i\left[E(r_m) - r_f\right]$$

Here:

$$\beta_i = \frac{\sigma_{im}}{\sigma_M^2}$$

β_i is often termed as the market risk of security i. However, the technically correct interpretation of β_i is: Contribution of security i to the variance of the market portfolio as a fraction of the total variance of the market portfolio.

Equilibrium Conditions

4. Risk premium on an individual security is a function of its covariance with the market.

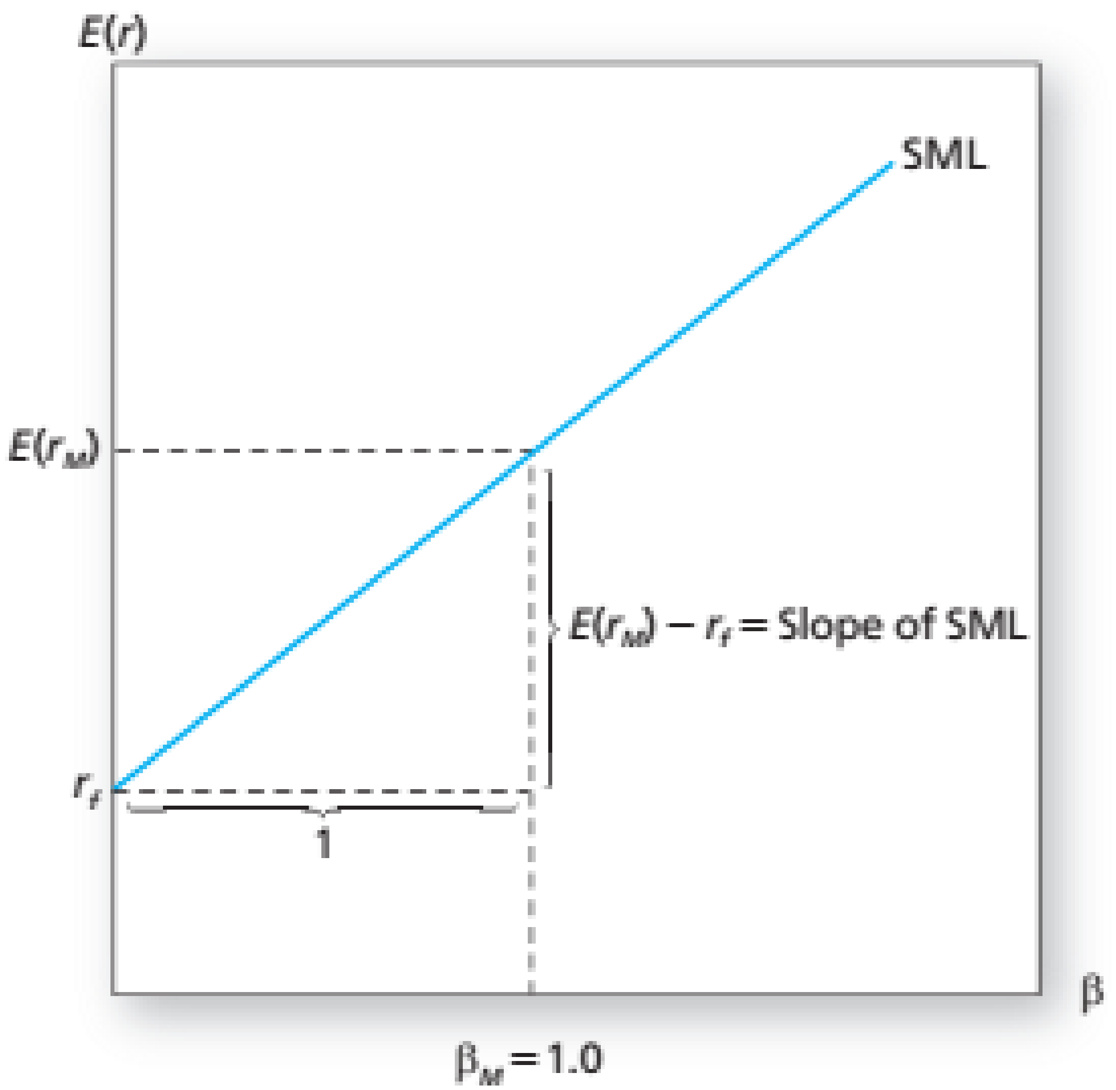
$$E(r_i) = r_f + \beta_i[E(r_m) - r_f]$$

- This expected return–beta (or mean-beta) relationship is the most familiar expression of the CAPM to practitioners.

- $E(r_i) - r_f$ is the required risk premium of security i and $E(r_i)$ often expressed simply as r_i is the (expected) required rate of return on security i.

- This mean-beta function gives us the Security Market Line or the SML: $E(r_i)$ as a function of β_i.

Security Market Line: SML



Sample Calculations for SML

Given:

$E(r_m) = 11\%, r_f = 3\%, \beta_x = 1.25, \beta_y = .6$

Compute required rates of return on securities *x* and *y*.

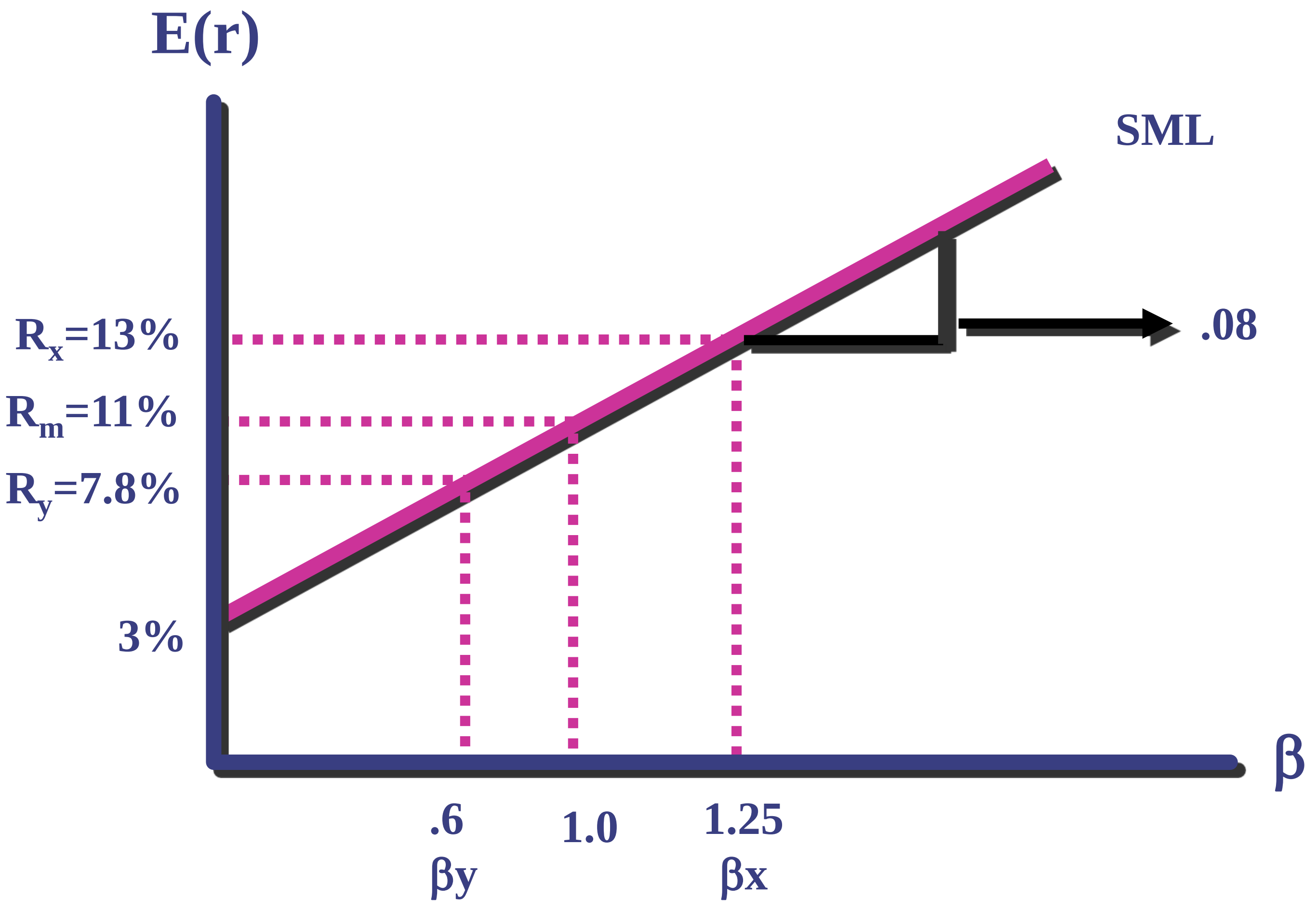
Use the SML equation:

$$E(r_i) = r_f + \beta_i[E(r_m) - r_f]$$

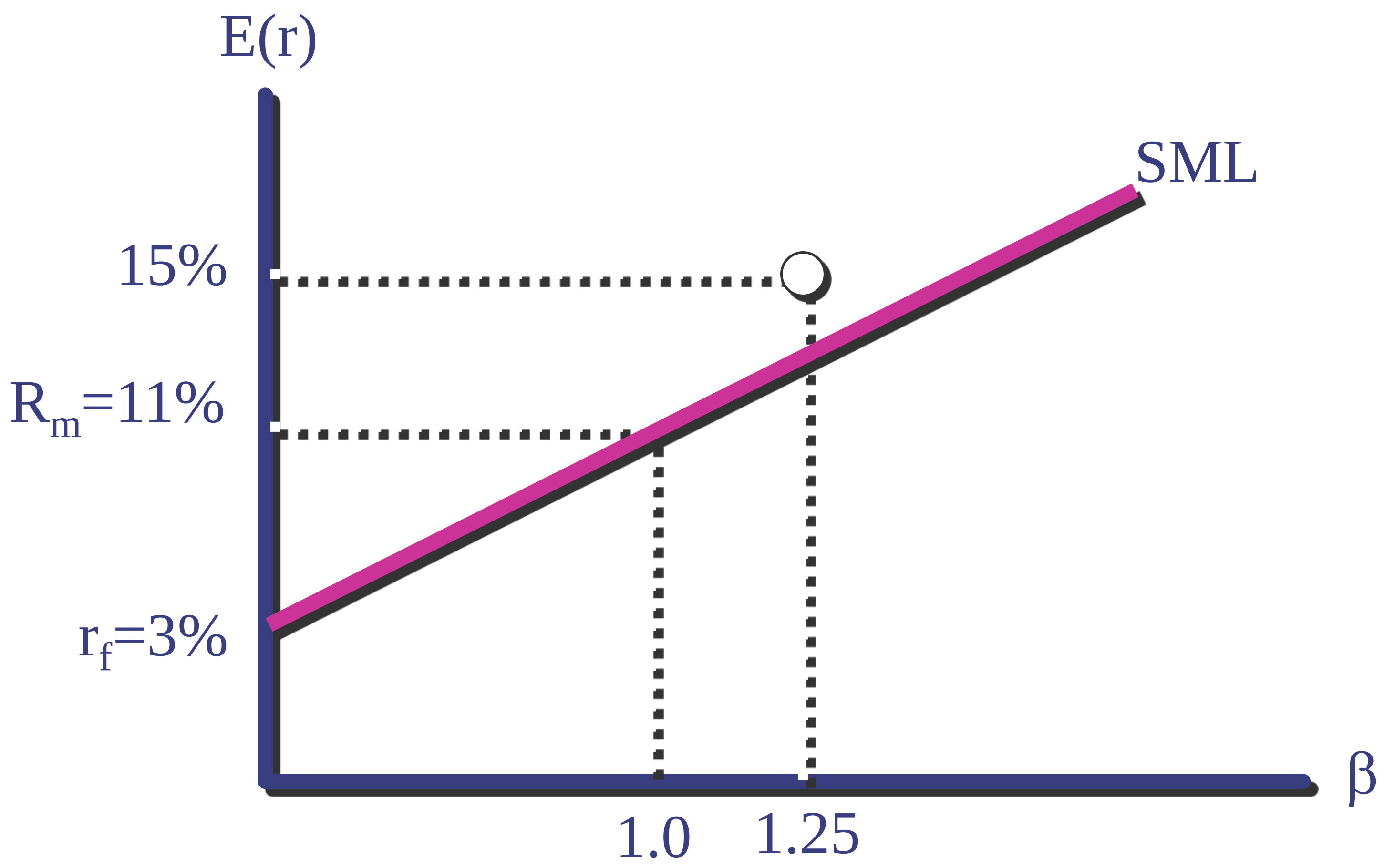
$$E(r_x) = 3\% + 1.25(11\% - 3\%) = 13\%$$

$$E(r_y) = 3\% + 0.60(11\% - 3\%) = 7.8\%$$

Graph of Sample Calculations



Disequilibrium Example



Disequilibrium Example

- Suppose a security with a β of 1.25 is offering expected return of 15%.
- According to SML, it should be 13%.
- Actual expected return – Fair expected return = 15%-13% = 2% = α
- The stock offering such return is called a positive-alpha stock.
- A positive-alpha security is an underpriced security offering too high of a rate of return for its level of risk.
- If positive-alpha securities exist in the market, then all investors should start buying the security and eventually the price should increase and the security's actual expected return should return to its fair level.

NPV and *irr*

- The (expected) required rate of return, r_i , on any security/investment can be predicted using the CAPM, e.g., for a single period investment horizon:

$$r_i = \frac{FV - PV}{PV}$$

$$\rightarrow PV r_i = FV - PV$$

$$\rightarrow PV(1 + r_i) = FV \rightarrow PV = \frac{FV}{1 + r_i}$$

- To evaluate any investment, we can compute the Net Present Value, NPV, of the investment, which is the sum of all the cash flows expected to be generated by the investment discounted to present value (assuming that the required rate of return r remains the same throughout the investment horizon):

$$NPV = -C_0 + \frac{C_1}{1 + r} + \frac{C_2}{(1 + r)^2} + \cdots + \frac{C_n}{(1 + r)^n}$$

NPV and *irr*

$$NPV = -C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_n}{(1+r)^n}$$

- The required rate of return 'r' in the above equation can be predicted by the CAPM.

- irr (internal rate of return) of the investment is the rate of return with which if we discount all the cash flows to their present values, then NPV will be equal to zero.

- Therefore, $NPV > 0 \rightarrow irr > r$ and $NPV < 0 \rightarrow irr < r$

- Decision criteria?

Practice Problems

- Chapter 9:

  1, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16