Chapter 10

Arbitrage Pricing Theory
Arbitrage

- **Arbitrage** - arises if an investor can construct a zero investment portfolio with a sure profit. Risk-less profit with zero initial outlay or investment.

- Since no investment is required, an investor can create large positions to secure large levels of profit.

- In efficient markets, profitable arbitrage opportunities will quickly disappear.
Arbitrage

Example: the same product is being transacted in two shops. The price in Shop A is Tk. 20 whereas in Shop B, the price is Tk. 22. Assume buying and selling prices are same. What will happen? How can you make risk-less profit with no initial outlay or investment? How will this arbitrage opportunity disappear in an efficient market?

The Law of One Price
## Arbitrage Example

<table>
<thead>
<tr>
<th>Stock</th>
<th>Current Price</th>
<th>$E(R)$</th>
<th>$\sigma$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$10</td>
<td>25%</td>
<td>29.58</td>
<td>$\rho_{AB} = -0.15$</td>
</tr>
<tr>
<td>B</td>
<td>$10</td>
<td>26%</td>
<td>33.91</td>
<td>$\rho_{BC} = -0.87$</td>
</tr>
<tr>
<td>C</td>
<td>$10</td>
<td>32.5%</td>
<td>48.15</td>
<td>$\rho_{AC} = -0.29$</td>
</tr>
<tr>
<td>D</td>
<td>$10</td>
<td>22.5%</td>
<td>8.58</td>
<td></td>
</tr>
</tbody>
</table>
### Arbitrage Portfolio

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$E(R)$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B, C</td>
<td>27.83</td>
<td>6.40</td>
</tr>
<tr>
<td>D</td>
<td>22.25</td>
<td>8.58</td>
</tr>
</tbody>
</table>
Arbitrage Action and Returns

Short 3 shares of D and buy 1 of A, B & C to form P.

You earn a higher rate on the investment than you pay on the short sale.
Stephen Ross developed the arbitrage pricing theory (APT) in 1976.

Ross’s APT relies on three key propositions:

1. Security returns can be described by a factor model.
2. There are sufficient securities to diversify away idiosyncratic risk.
3. Well-functioning security markets do not allow for the persistence of arbitrage opportunities.
Arbitrage Pricing Theory (APT)

- APT is based on the law of one price
- It does not rely on mean-variance assumption (as the CAPM does)
- It assumes that asset returns are **linearly** related to a set of indexes. Each index represents a factor that influences the return on an asset.

\[ R_i = E(R_i) + \sum_{j=1}^{n} \beta_{ij} F_j + e_i \]

- \(R_i\) denotes excess return of security \(i\).
- \(F_j\) = deviation of factor \(j\) from its common value
- \(e_i\) denotes firm-specific disturbance, which is uncorrelated with \(F_j\)
- For a well-diversified portfolio: \(e_i\) approaches zero
Arbitrage Pricing Theory (APT)

- For example:

\[ R_i = E(R_i) + \beta_{iGDP}GDP + \beta_{iIR}IR + e_i \]

Here, GDP denotes unanticipated growth in GDP and IR denotes unexpected change in interest rate.

- Market participants develop expectations about the values of the sensitivities (betas).

- They will buy and sell securities so that, given the law of one price, securities affected equally by the same factors will have equal expected returns.

- This buying and selling is the arbitrage process, which determines the prices of securities.
Example 10.2  Risk Assessment Using Multifactor Models

Suppose we estimate the two-factor model in Equation 10.2 for Northeast Airlines and find the following result:

\[ R = 0.133 + 1.2(GDP) - 0.3(IR) + e \]

This tells us that, based on currently available information, the expected excess rate of return for Northeast is 13.3%, but that for every percentage point increase in GDP beyond current expectations, the return on Northeast shares increases on average by 1.2%, while for every unanticipated percentage point that interest rates increases, Northeast’s shares fall on average by .3%.
Figure 10.1 Excess returns as a function of the systematic factor. **Panel A**, well-diversified portfolio A. **Panel B**, single stock (S).
Disequilibrium Example

Figure 10.3 An arbitrage opportunity
Disequilibrium Example

- Short Portfolio C

- Use funds to construct an equivalent risk higher return Portfolio D.
  - D is comprised of A & Risk-Free Asset (where proportion of funds in A is 50%).

- Arbitrage profit of 1%
APT with Market Index Portfolio

\[ E(r) \]

\[ r_f \]

\[ \beta \text{ (with respect to the Market Index)} \]

\[ [E(r_M) - r_f] \]

Market Risk Premium
The CAPM is a special case of APT that would result if the single common factor affecting all security returns was the return on the market portfolio.

APT is more general, or robust, than the CAPM. It is based on less restrictive assumptions.

APT does not identify either the number or the definition of the factors affecting returns. These have to be empirically determined by fitting a factor model to returns.

The CAPM is a well-specified model, where the parameters of the model are spelled out up front. However, it relies on the Market Portfolio, which is in principle unmeasurable. APT is more general in that it gets to an expected return and beta relationship without the assumption of the market portfolio.